Walk-on-Spheres Algorithm for Solving Boundary-Value Problems with Continuity Flux Conditions

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Summary. We consider boundary-value problem for elliptic equations with constant coefficients related through the continuity conditions on the boundary between the domains. To take into account conditions involving the solution's normal derivative, we apply a new mean-value relation written down at a boundary point. This integral relation is exact and provides a possibility to get rid of the bias caused by usually used finite-difference approximation. Randomization of this mean-value relation makes it possible to continue simulating walk-on-spheres trajectory after it hits the boundary. We prove the convergence of the algorithm and determine its rate. In conclusion, we present the results of some model computations.

1 Statement of the Problem

We consider the boundary-value problem for a function, $u(x)$, that satisfies different elliptic equations with constant coefficients inside a bounded simple-connected domain, $G_i \subset \mathbb{R}^3$, and in its exterior, $G_e = \mathbb{R}^3 \setminus \overline{G_i}$.

Denote, for convenience, by $u_i(x)$ and $u_e(x)$ the restrictions of function $u(x)$ to G_i and G_e, respectively. Let the first function satisfy the Poisson equation

$$\epsilon_i \Delta u_i = -\rho, \tag{1}$$

and the second one satisfy the linearized Poisson-Boltzmann equation

$$\epsilon_e \Delta u_e - \epsilon_e \kappa^2 u_e = 0. \tag{2}$$

The continuity conditions on the piecewise smooth boundary, Γ, relate limiting values of solutions, and their fluxes as well:

$$u_i(y) = u_e(y)\ ,\ \epsilon_i \frac{\partial u_i}{\partial n}(y) = \epsilon_e \frac{\partial u_e}{\partial n}(y)\ ,\ y \in \Gamma. \tag{3}$$